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General Rotating Five Dimensional Black Holes of Toroidally Compactified Heterotic String

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Abstract

We present the most general rotating black hole solution of five-dimensional $N = 4$ superstring vacua that conforms to the "no hair theorem". It is chosen to be parameterized in terms of massless fields of the toroidally compactified heterotic string. The solutions are obtained by performing a subset of $O(8,24)$ transformations, *i.e.*, symmetry transformations of the effective three-dimensional action for stationary solutions, on the five-dimensional (neutral) rotating solution parameterized by the mass m and two rotational parameters l_1 and l_2. The explicit form of the generating solution is determined by three $SO(1,1) \subset O(8,24)$ boosts, which specify two electric charges $Q^{(1)}_1$, $Q^{(2)}_2$ of the Kaluza-Klein and two-form $U(1)$ gauge fields associated with the same compactified direction, and the charge Q (electric charge of the vector field, whose field strength is dual to the field strength of the five-dimensional two-form field). The general solution, parameterized by 27 charges, two rotational parameters and the ADM mass compatible with the Bogomol'nyi bound, is obtained by imposing $[SO(5) \times SO(21)]/[SO(4) \times SO(20)] \subset O(5,21)$ transformations, which do not affect the five-dimensional space-time. We also analyze the deviations from the BPS-saturated limit.

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I. INTRODUCTION

Recently, certain black holes in string theory have attracted an overwhelming attention due to the fact that it has now become feasible to address their thermal properties in terms of microscopic degrees of freedom associated with the D-brane configuration [1], describing a particular black hole. In particular, the microscopic entropy of certain five-dimensional BPS-saturated static [2,3] and rotating [4] black holes as well as that for specific infinitesimal deviations from the BPS-saturated limit for static [5] and rotating [6] six-dimensional black strings (or five-dimensional black holes upon dimensionally reducing on S^1) have been provided [2].

Interestingly, the "D-brane technology" allows one to calculate the microscopic entropy of certain types of black holes whose explicit classical configurations have either not been constructed, yet, or have been constructed only for a specific assignment of charges. The purpose of this paper is to fill such a void for the 5-dimensional rotating black hole solutions. Namely, we shall present an explicit form of the generating solution for the general rotating black hole solution of five-dimensional $N = 4$ superstring vacua, that conforms to the "no hair theorem" [3] [12]. We choose to parameterize the general solution in terms of massless fields of the heterotic string compactified on a five-torus (T^5) [4].

We use a generating technique by performing a subset of $O(8, 24)$ transformations, *i.e.*, symmetry transformations of the effective three-dimensional action for stationary solutions, on the five-dimensional (neutral) rotating solution parameterized by the mass m and two rotating parameters l_1 and l_2. The explicit form of the generating solution is determined by three $SO(1, 1) \subset O(8, 24)$ boosts δ_{e1}, δ_{e2}, and δ_e, which specify respectively the two electric charges $Q_1^{(1)}$, $Q_1^{(2)}$ of the two $U(1)$ gauge fields, *i.e.*, the Kaluza-Klein $A^{(1)}_{\mu 1}$ and the two-form $A^{(2)}_{\mu 1}$ gauge fields associated with the first compactified direction, and the charge Q, *i.e.*, the

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[1]For a review on D-brane physics see Ref. [1].

[2]Recently, the microscopic entropy in terms of the degrees of freedom of the corresponding D-brane configurations for certain (four-parameter) four-dimensional BPS-saturated black holes has been given [7,8], while an earlier complementary approach was initiated in Ref. [9] with further elaboration in Refs. [10,11].

[3]The content of the conjecture no-hair theorem states that the ADM mass, the angular momenta and a set of conserved (electric and magnetic) charges uniquely specifies classical black hole solution. In the case under consideration in this paper, the black hole solutions are uniquely parameterized by the ADM mass (which can be traded for the non-extremality parameter $m > 0$), two angular momenta and 27 (conserved) electric charges.

[4]Equivalent parameterization is possible (due to string-string duality) in terms of fields of Type IIA compactified on $K3 \times S^1$ or T-dualized Type IIB string. In the latter cases the generating solution has a map onto Ramond-Ramond (RR) charges and thus an interpretation in terms of D-brane configurations.

electric charge of the vector field, whose field strength is dual to the field strength $H_{\mu\nu\rho}$ of the five-dimensional two form field $B_{\mu\nu}$. The general solution, parameterized by 27 charges, two rotational parameters and the ADM mass compatible with the Bogomol'nyi bound, is obtained by imposing on the generating solution $[SO(5)\times SO(21)]/[SO(4)\times SO(20)]\subset O(5,21)$ transformations, which do not affect the five-dimensional space-time. Here the $O(5,21)$ symmetry corresponds to the T-duality symmetry of the five-dimensional toroidally compactified heterotic string vacuum.

Our results reproduce the generating solution for the BPS-saturated static [2,11,3] and rotating [4,11] black holes (as well as specific deviations [5,6] from the BPS limit) as special examples. The mass formula and the area of the horizon for the general BPS-saturated black hole at generic points of the $N=4$ moduli and coupling space is written in terms of one rotational parameter, 27 (conserved) quantized charges as well as the arbitrary asymptotic values of 115 moduli fields and the dilaton. Again, the area of the horizon is independent of the moduli and the dilaton couplings.

The paper is organized as follows. In Section II we write down the explicit form of the toroidally compactified effective heterotic string action in D-dimensions, with the emphasis on five-dimensional action and three-dimensional effective action suitable for describing the stationary solutions in higher dimensions. In Section III we give the explicit form of the generating solution, specified by six parameters (mass, three charges and two rotational parameters) and give a subset of T-duality transformations which allow one to write the general solution in terms of the mass, 27 (quantized) charges and two rotational parameters, as well as arbitrary asymptotic values of moduli and dilaton couplings. In Section IV we concentrate on the BPS-saturated solution, and properties of the deviations from the BPS limit. In particular we address the mass formula and the area of the horizon.

II. ACTION OF HETEROTIC STRING ON TORI

In this chapter, we shall discuss effective actions of the massless bosonic fields of heterotic string compactified on tori. The starting point is the following zero-slope limit effective action of the heterotic string with zero mass fields given by the graviton $\hat{G}_{MN}$, the two form field $\hat{B}_{MN}$, $U(1)^{16}$ part $\hat{A}^I_M$ of the ten-dimensional gauge group, and the dilaton $\hat{\Phi}$ $(M,N=0,1,...,9,\ I=1,...,16)$ [5]:

$$\mathcal{L}=\frac{1}{16\pi G_{10}}\sqrt{-\hat{G}}\,e^{-\hat{\Phi}}[\mathcal{R}_{\hat{G}}+\hat{G}^{MN}\partial_M\hat{\Phi}\partial_N\hat{\Phi}-\tfrac{1}{12}\hat{H}_{MNP}\hat{H}^{MNP}-\tfrac{1}{4}\hat{F}^I_{MN}\hat{F}^{I\,MN}], \tag{1}$$

where G_{10} is the ten-dimensional Newton's constant, which in this paper is chosen to be $8\pi^6$, $\hat{G}\equiv\det\hat{G}_{MN}$, $\mathcal{R}_{\hat{G}}$ is the Ricci scalar of the metric $\hat{G}_{MN}$, $\hat{F}^I_{MN}=\partial_M\hat{A}^I_N-\partial_N\hat{A}^I_M$ and $\hat{H}_{MNP}=\partial_M\hat{B}_{NP}-\frac{1}{2}\hat{A}^I_M\hat{F}^I_{NP}+\text{cyc. perms.}$ are the field strengths of $\hat{A}^I_M$ and $\hat{B}_{MN}$, respectively. We choose the mostly positive signature convention $(-++\cdots+)$ for the metric $\hat{G}_{MN}$.

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[5] We follow the convention spelled out in Refs. [13,14].

A. Effective Action of Heterotic String Compactified on a $(10-D)$-Torus

The compactification of the extra $(10-D)$ spatial coordinates on a $(10-D)$-torus can be achieved by choosing the following Abelian Kaluza-Klein Ansatz for the ten-dimensional metric

$$\hat{G}_{MN} = \begin{pmatrix} e^{a\varphi}g_{\mu\nu} + G_{mn}A^{(1)\,m}_{\mu}A^{(1)\,n}_{\nu} & A^{(1)\,m}_{\mu}G_{mn} \\ A^{(1)\,n}_{\nu}G_{mn} & G_{mn} \end{pmatrix}, \tag{2}$$

where $A^{(1)\,m}_{\mu}$ ($\mu = 0, 1, ..., D-1$; $m = 1, ..., 10-D$) are D-dimensional Kaluza-Klein $U(1)$ gauge fields, $\varphi \equiv \hat{\Phi} - \frac{1}{2}\ln\det G_{mn}$ is the D-dimensional dilaton field, and $a \equiv \frac{2}{D-2}$. Then, the affective action is specified by the following massless bosonic fields: the (Einstein-frame) graviton $g_{\mu\nu}$, the dilaton φ, $(36-2D)$ $U(1)$ gauge fields $\mathcal{A}^i_{\mu} \equiv (A^{(1)\,m}_{\mu}, A^{(2)}_{\mu\,m}, A^{(3)\,I}_{\mu})$ defined as $A^{(2)}_{\mu\,m} \equiv \hat{B}_{\mu m} + \hat{B}_{mn}A^{(1)\,n}_{\mu} + \frac{1}{2}\hat{A}^I_m A^{(3)\,I}_{\mu}$, $A^{(3)\,I}_{\mu} \equiv \hat{A}^I_{\mu} - \hat{A}^I_m A^{(1)\,m}_{\mu}$, the two-form field $B_{\mu\nu} \equiv \hat{B}_{\mu\nu} - \hat{B}_{mn}A^{(1)\,m}_{\mu}A^{(1)\,n}_{\nu} - \frac{1}{2}(A^{(1)\,m}_{\mu}A^{(2)}_{\nu\,m} - A^{(1)\,m}_{\nu}A^{(2)}_{\mu\,m})$, and the following symmetric $O(10-D, 26-D)$ matrix of the scalar fields (moduli):

$$M = \begin{pmatrix} G^{-1} & -G^{-1}C & -G^{-1}a^T \\ -C^TG^{-1} & G + C^TG^{-1}C + a^Ta & C^TG^{-1}a^T + a^T \\ -aG^{-1} & aG^{-1}C + a & I + aG^{-1}a^T \end{pmatrix}, \tag{3}$$

where $G \equiv [\hat{G}_{mn}]$, $C \equiv [\frac{1}{2}\hat{A}^{(I)}_m\hat{A}^{(I)}_n + \hat{B}_{mn}]$ and $a \equiv [\hat{A}^I_m]$ are defined in terms of the internal parts of ten-dimensional fields. Then the effective D-dimensional effective action takes the form:

$$\begin{aligned}\mathcal{L} = \frac{1}{16\pi G_D}\sqrt{-g}[&\mathcal{R}_g - \frac{1}{(D-2)}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{8}g^{\mu\nu}\mathrm{Tr}(\partial_\mu ML\partial_\nu ML) - \frac{1}{12}e^{-2a\varphi}g^{\mu\mu'}g^{\nu\nu'}g^{\rho\rho'}H_{\mu\nu\rho}H_{\mu'\nu'\rho'} \\ &- \frac{1}{4}e^{-a\varphi}g^{\mu\mu'}g^{\nu\nu'}\mathcal{F}^i_{\mu\nu}(LML)_{ij}\mathcal{F}^j_{\mu'\nu'}],\end{aligned} \tag{4}$$

where G_D is the D-dimensional Newton's constant [6], $g \equiv \det g_{\mu\nu}$, $\mathcal{R}_g$ is the Ricci scalar of $g_{\mu\nu}$, $\mathcal{F}^i_{\mu\nu} = \partial_\mu\mathcal{A}^i_\nu - \partial_\nu\mathcal{A}^i_\mu$ are the $U(1)^{36-2D}$ gauge field strengths, and $H_{\mu\nu\rho} \equiv (\partial_\mu B_{\nu\rho} - \frac{1}{2}\mathcal{A}^i_\mu L_{ij}\mathcal{F}^j_{\nu\rho})$ + cyc. perms. ofμ, ν, ρ is the field strength of the two-form field $B_{\mu\nu}$. Here, in the above we have chosen a metric L of $O(10-D, 26-D)$ group to be:

$$L = \begin{pmatrix} 0 & I_{10-D} & 0 \\ I_{10-D} & 0 & 0 \\ 0 & 0 & I_{26-D} \end{pmatrix}, \tag{5}$$

where I_n denotes the $n \times n$ identity matrix. The D-dimensional effective action (4) is invariant under the $O(10-D, 26-D)$ transformations (T-duality) [13,14]:

$$M \to \Omega M \Omega^T, \quad \mathcal{A}^i_\mu \to \Omega_{ij}\mathcal{A}^j_\mu, \quad g_{\mu\nu} \to g_{\mu\nu}, \quad \varphi \to \varphi, \quad B_{\mu\nu} \to B_{\mu\nu}, \tag{6}$$

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[6] The Newton's constant G_D in D-dimensions can be expressed in terms of the 10-dimensional one G_{10} as $G_{10} = (2\pi\sqrt{\alpha'})^{10-D}G_D$, where in this paper we choose $\alpha' = 1$.

where Ω is an $O(10-D, 26-D)$ invariant matrix, *i.e.*, $\Omega^T L \Omega = L$.

In particular for $D=5$ the effective action is specified by the graviton, 116 scalar fields (115 moduli fields in the matrix M and the dilaton φ), 26 $U(1)$ gauge fields, and the field strength $H_{\mu\nu\rho}$ of the two form field $B_{\mu\nu}$. By the duality transformation:

$$H^{\mu\nu\rho} = -\frac{e^{4\varphi/3}}{2!\sqrt{-g}} \varepsilon^{\mu\nu\rho\lambda\sigma} F_{\lambda\sigma}, \tag{7}$$

$H_{\mu\nu\rho}$ can be related to the field strength $F_{\mu\nu}$ of the gauge field A_μ, and therefore in five-dimensions black hole solution carries extra $U(1)$ charge associated with the dual of the three-form field strength.

The T-duality symmetry of the effective action is $O(5,21)$. The equations of motion and Bianchi identities are also invariant under the $SO(1,1)$ symmetry [7]:

$$\varphi \to \varphi + \beta, \quad H_{\mu\nu\rho} \to \mathrm{e}^{-2\beta/3} H_{\mu\nu\rho}, \quad \mathcal{F}^i_{\mu\nu} \to \mathrm{e}^{+\beta/3} \mathcal{F}^i_{\mu\nu}, \quad M \to M, \quad g_{\mu\nu} \to g_{\mu\nu}, \tag{8}$$

where β is an $SO(1,1)$ boost parameter. Such an $SO(1,1)$ symmetry can be used to set the asymptotic value of the three-dimensional dilaton to be $\varphi_\infty = 0$.

At the quantum level, the parameters of both $O(5,21)$ and $SO(1,1)$ symmetry transformations become integer-valued.

B. Three-Dimensional Effective Action

The generating five-dimensional *charged* rotating solution is obtained by imposing $SO(1,1) \subset O(8,24)$ boost transformations on the five-dimensional neutral solution. Here $O(8,24)$ is the symmetry of the three-dimensional effective action of the heterotic string theory with Killing coordinates (which we choose to be the time coordinate t, one of the angles of the rotations (which we choose to be ψ), and the internal coordinates of the five-dimensional space-time compactified on a seven-torus. The $O(8,24)$ contains as subsets three-dimensional $O(7,23)$ T-duality symmetry discussed in the previous section and four-dimensional $SL(2)$ S-duality symmetry. Such an enhancement of the duality symmetry is possible due to the fact that in three-dimensions the field strength of a one-form field is dual to a scalar as discussed below. Such an action is given by:

$$\mathcal{L} = \tfrac{1}{4}\sqrt{-h}\,[\mathcal{R}_h + \tfrac{1}{8} h^{\bar\mu\bar\nu} \mathrm{Tr}(\partial_{\bar\mu}\mathcal{M}\mathbf{L}\partial_{\bar\nu}\mathcal{M}\mathbf{L})], \tag{9}$$

where $h \equiv \det h_{\bar\mu\bar\nu}$ and $\mathcal{R}_h$ is the Ricci scalar of the three-dimensional metric $h_{\bar\mu\bar\nu}$ ($\bar\mu, \bar\nu = r, \theta, \phi$). Here, we use "bars" to denote fields in three-dimensions. $\mathcal{M}$ is a symmetric $O(8,24)$ matrix of three-dimensional scalar fields defined as

$$\mathcal{M} = \begin{pmatrix} \bar M - e^{2\bar\varphi}\psi\psi^T & e^{2\bar\varphi}\psi & \bar M \bar L \psi - \frac{1}{2} e^{2\bar\varphi}\psi(\psi^T \bar L \psi) \\ e^{2\bar\varphi}\psi^T & -e^{2\bar\varphi} & \frac{1}{2} e^{2\bar\varphi}\psi^T \bar L \psi \\ \psi^T \bar L \bar M - \frac{1}{2} e^{2\bar\varphi}\psi^T(\psi^T \bar L \psi) & \frac{1}{2} e^{2\bar\varphi}\psi^T \bar L \psi & -e^{-2\bar\varphi} + \psi^T \bar L \bar M \bar L \psi - \frac{1}{4} e^{2\bar\varphi}(\psi^T \bar L \psi)^2 \end{pmatrix}, \tag{10}$$

[7]For a review of the supergravity duality groups in different dimensions see, *e.g.*, Ref. [15].

where $\bar{M}$ is the $O(7,23)$ symmetric moduli field and $\bar{\varphi}$ is the three-dimensional dilaton. Here, $\psi \equiv [\psi^i]$ is the scalars dual to the three-dimensional $U(1)$ gauge fields $\bar{\mathcal{A}}^i_\mu$ [16]:

$$\sqrt{-h} e^{-2\bar{\varphi}} h^{\bar{\mu}\bar{\mu}'} h^{\bar{\nu}\bar{\nu}'} (\bar{M}\bar{L})_{\bar{i}\bar{j}} \bar{\mathcal{F}}^{\bar{j}}_{\bar{\mu}'\bar{\nu}'} = \epsilon^{\bar{\mu}\bar{\nu}\bar{\rho}} \partial_{\bar{\rho}} \psi^{\bar{i}}. \tag{11}$$

The action is manifestly invariant under the $O(8,24)$ transformations:

$$\mathcal{M} \to \mathbf{\Omega} \mathcal{M} \mathbf{\Omega}^T, \qquad h_{\bar{\mu}\bar{\nu}} \to h_{\bar{\mu}\bar{\nu}}, \tag{12}$$

where $\mathbf{\Omega} \in O(8,24)$, *i.e.*,

$$\mathbf{\Omega} \mathbf{L} \mathbf{\Omega}^T = \mathbf{L}, \quad \mathbf{L} = \begin{pmatrix} \bar{L} & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{13}$$

III. GENERAL SOLUTION

In this Section we briefly spell out the solution generating technique [8], and the explicit sequence of symmetry transformations applied on the five-dimensional neutral rotating solution. Then, we give the explicit form of the generating solution. In the last subsection we give explicit subset of five-dimensional T-duality ($O(5,21)$) transformations.

A. Solution Generating Technique

An arbitrary asymptotic value of the scalar field matrix $\mathcal{M}$ can be transformed into the form

$$\mathcal{M}_\infty = \mathrm{diag}(-1, \overbrace{1, ..., 1}^{6}, -1, \overbrace{1, ..., 1}^{22}, -1, -1) \equiv I_{4,28} \tag{14}$$

by imposing an $O(8,24)$ transformation, *i.e.*, $\mathcal{M}_\infty \to \mathbf{\Omega} \mathcal{M}_\infty \mathbf{\Omega}^T = I_{4,28}$ ($\mathbf{\Omega} \in O(8,24)$). Thus, without loss of generality we can confine the analysis by choosing the asymptotic value of the form to be $\mathcal{M}_\infty = I_{4,28}$ and, then, obtain the solutions with arbitrary values of $\mathcal{M}_\infty$ by undoing the above constant $O(8,24)$ transformation [9]. Then, the subset of $O(8,24)$ transformations that preserves the asymptotic value $\mathcal{M}_\infty = I_{4,28}$ is $SO(8) \times SO(24)$.

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[8] A general approach of generating charged solutions from the charge neutral solutions within the toroidally compactified heterotic string is developed in Ref. [17].

[9] In the case of the five-dimensional configurations with the asymptotically flat five-dimensional space-time metric, the subset of such $O(8,24)$ transformations that preserves the asymptotic flatness of the five-dimensional space-time corresponds to the constant $O(5,21) \times SO(1,1)$ transformations (see Eqs. (6) and (8)), which allow one to bring the asymptotic values of the moduli matrix $M_\infty = I_{26}$ and the dilaton field $\varphi_\infty = 0$ into arbitrary asymptotic values. At the same time the physical charges are written in terms of *quantized* charges, which are "dressed" by the asymptotic values of the moduli and dilaton.

The starting point for obtaining the generating five-dimensional charged rotating solution is the five-dimensional neutral solution with two rotational parameters $l_{1,2}$ [18]. We choose to write it in the following form:

$$\begin{aligned} ds^2 &= -\frac{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta-2m}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}dt^2+\frac{r^2(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)}{(r^2+l_1^2)(r^2+l_2^2)-2mr^2}dr^2 \\ &+(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)d\theta^2+\frac{4ml_1l_2\sin^2\theta\cos^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}d\phi d\psi \\ &+\frac{\sin^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}[(r^2+l_1^2)(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)+2ml_1^2\sin^2\theta]d\phi^2 \\ &+\frac{\cos^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}[(r^2+l_2^2)(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)+2ml_2^2\cos^2\theta]d\psi^2 \\ &-\frac{4ml_1\sin^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}dtd\phi-\frac{4ml_2\cos^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}dtd\psi. \end{aligned} \tag{15}$$

To apply $SO(1,1)\subset O(8,22)$ boosts on the Kerr solution (15), one has to rewrite (15) in terms of the three-dimensional fields in (9) by performing the standard Kaluza-Klein dimensional reduction, using the the corresponding five-dimensional Ansatz of (2), and, then, by performing the duality transformation (11):

$$\begin{aligned} h_{\bar\mu\bar\nu}dx^{\bar\mu\bar\nu} &= \frac{[(r^2+l_2^2)(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta-2m)+2ml_2^2\cos^2\theta]r^2\cos^2\theta}{r^2(r^2+l_1^2+l_2^2-2m)+l_1^2l_2^2}dr^2 \\ &+[(r^2+l_2^2)(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta-2m)+2ml_2^2\cos^2\theta]\cos^2\theta d\theta^2 \\ &+[r^2(r^2+l_1^2+l_2^2-2m)+l_1^2l_2^2]\cos^2\theta\sin^2\theta d\phi^2, \\ \psi_8 &= \frac{2ml_1\cos^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}, \quad \psi_9=-\frac{2ml_1l_2\cos^4\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}, \\ \bar g_{11} &= -\frac{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta-2m}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}, \quad \bar g_{12}=-\frac{2ml_2\cos^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}, \\ \bar g_{22} &= \frac{\cos^2\theta[(r^2+l_2^2)(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)+2ml_2^2\cos^2\theta]}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}, \\ e^{-2\bar\varphi} &= \frac{(r^2+l_2^2)(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta-2m)+2ml_2^2\cos^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}\cos^2\theta. \end{aligned} \tag{16}$$

We shall proceed with the following sequence of the symmetry transformations on (16). In order to obtain the explicit form of the generating charged rotating solution, we shall apply three necessary $SO(1,1)\subset O(8,24)$ boost transformations on the neutral rotating solution (16), which would yield the generating solution specified by the mass, *three* charge parameters, and two rotational parameters. We then apply on the generating solution the $[SO(5)\times SO(21)]/[SO(4)\times SO(20)]$ transformations, *i.e.*, the subsets of T-duality transformations, which do not affect the five-dimensional space-time (and preserve the asymptotic values $M_\infty=I_{26}$ and $\varphi_\infty=0$), however, they introduce 4+20 additional charge parameters. Thus, one in turn obtains the most general configuration specified by the mass, 27 electric charges and two rotational parameters.

B. Generating Solution

The generating five-dimensional charged rotating solution can be obtained by applying the following three $SO(1,1) \subset O(8,24)$ boosts on the three-dimensional scalar matrix $\mathcal{M}$, specified by the neutral rotating solution (16). The boost transformation $\mathbf{\Omega}_e$ has the following form:

$$\mathbf{\Omega}_e \equiv \begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cosh\delta_e & \cdot & \cdot & \cdot & -\sinh\delta_e & \cdot \\ \cdot & \cdot & I_6 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cosh\delta_e & \cdot & \cdot & \sinh\delta_e \\ \cdot & \cdot & \cdot & \cdot & I_{21} & \cdot & \cdot \\ \cdot & -\sinh\delta_e & \cdot & \cdot & \cdot & \cosh\delta_e & \cdot \\ \cdot & \cdot & \cdot & \sinh\delta_e & \cdot & \cdot & \cosh\delta_e \end{pmatrix}, \tag{17}$$

where the dots denote the corresponding zero entries and $\mathbf{\Omega}_{e1}$ has the analogous form with the non-trivial entries (with positive signs in front of sinh's) in the $\{8th, 10th\}$ and (with negative signs in front of sinh's) in the $\{1st, 3rd\}$ columns and rows. The third transformation $\mathbf{\Omega}_{e2}$ has the non-trivial entries (with positive signs in front of sinh's) in the $\{3rd, 8th\}$ and (with negative signs in front of sinh's) in the $\{1st, 10th\}$ columns and rows.

The above three boosts $\delta_e, \delta_{e1}, \delta_{e2}$ induce the respective three $U(1)$ charges: Q is the (electric) charge of the $U(1)$ gauge field strength dual to the field strength $H_{\mu\nu\rho}$ of the two-form field $B_{\mu\nu}$, while $Q_1^{(1)}$ and $Q_1^{(2)}$ are the (electric) charges of $A^{(1)}_{\mu\,1}$ gauge field (Kaluza-Klein gauge field, associated with the first compactified direction), $A^{(2)}_{\mu\,1}$ gauge field (gauge field arising from the ten-dimensional two-form field, associated with the first compactified direction).

The final expression of the solution in terms of the (non-trivial) five-dimensional bosonic fields is of the form [10]:

$$\begin{aligned}
g_{11} &= \frac{r^2+2m\sinh^2\delta_{e1}+l_1^2\cos^2\theta+l_2^2\sin^2\theta}{r^2+2m\sinh^2\delta_{e2}+l_1^2\cos^2\theta+l_2^2\sin^2\theta},\\
e^{2\varphi} &= \frac{(r^2+2m\sinh^2\delta_e+l_1^2\cos^2\theta+l_2^2\sin^2\theta)^2}{(r^2+2m\sinh^2\delta_{e2}+l_1^2\cos^2\theta+l_2^2\sin^2\theta)(r^2+2m\sinh^2\delta_{e1}+l_1^2\cos^2\theta+l_2^2\sin^2\theta)},\\
A^{(1)}_{t\,1} &= \frac{m\cosh\delta_{e1}\sinh\delta_{e1}}{r^2+2m\sinh^2\delta_{e1}+l_1^2\cos^2\theta+l_2^2\sin^2\theta},\\
A^{(1)}_{\phi\,1} &= m\sin^2\theta\frac{l_1\sinh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e-l_2\cosh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e}{r^2+2m\sinh^2\delta_{e1}+l_1^2\cos^2\theta+l_2^2\sin^2\theta},\\
A^{(1)}_{\psi\,1} &= m\cos^2\theta\frac{l_1\cosh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e-l_2\sinh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e}{r^2+2m\sinh^2\delta_{e1}+l_1^2\cos^2\theta+l_2^2\sin^2\theta},\\
A^{(2)}_{t\,1} &= \frac{m\cosh\delta_{e2}\sinh\delta_{e2}}{r^2+2m\sinh^2\delta_{e2}+l_1^2\cos^2\theta+l_2^2\sin^2\theta},
\end{aligned}$$

[10]The five-dimensional Newton's constant is taken to be $G_N^{D=5} = \frac{\pi}{4}$ and we follow the convention of, *e.g.*, Ref. [18] for the definitions of the ADM mass, charges and angular momenta.

$$
\begin{aligned}
A^{(2)}_{\phi\,1} &= m\sin^2\theta\frac{l_1\cosh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e - l_2\sinh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e}{r^2+2m\sinh^2\delta_{e2}+l_1^2\cos^2\theta+l_2^2\sin^2\theta},\\
A^{(2)}_{\psi\,1} &= m\cos^2\theta\frac{l_1\sinh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e - l_2\cosh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e}{r^2+2m\sinh^2\delta_{e2}+l_1^2\cos^2\theta+l_2^2\sin^2\theta},\\
B_{t\phi} &= -2m\sin^2\theta(l_1\sinh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e - l_2\cosh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e)(r^2+l_1^2\cos^2\theta\\
&\quad +l_2^2\sin^2\theta+m\sinh^2\delta_{e1}+m\sinh^2\delta_{e2})/[(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta+2m\sinh^2\delta_{e1})\\
&\quad \times(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta+2m\sinh^2\delta_{e2})],\\
B_{t\psi} &= -2m\cos^2\theta(l_2\sinh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e - l_1\cosh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e)(r^2+l_1^2\cos^2\theta\\
&\quad +l_2^2\sin^2\theta+m\sinh^2\delta_{e1}+m\sinh^2\delta_{e2})/[(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta+2m\sinh^2\delta_{e1})\\
&\quad \times(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta+2m\sinh^2\delta_{e2})],\\
B_{\phi\psi} &= \frac{2m\cosh\delta_e\sinh\delta_e\cos^2\theta\sin^2\theta(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta+m\sinh^2\delta_{e1}+m\sinh^2\delta_{e2})}{(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta+2m\sinh^2\delta_{e1})(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta+2m\sinh^2\delta_{e2})},\\
ds^2_E &= \bar{\Delta}^{\frac{1}{3}}\Bigg[-\frac{(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta-2m)}{\bar{\Delta}}dt^2\\
&\quad + \frac{r^2}{(r^2+l_1^2)(r^2+l_2^2)-2mr^2}dr^2+d\theta^2+\frac{4m\cos^2\theta\sin^2\theta}{\bar{\Delta}}[l_1l_2\{(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)\\
&\quad - 2m(\sinh^2\delta_{e1}\sinh^2\delta_{e2}+\sinh^2\delta_e\sinh^2\delta_{e1}+\sinh^2\delta_e\sinh^2\delta_{e2})\}+2m\{(l_1^2+l_2^2)\\
&\quad \times\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e - 2l_1l_2\sinh^2\delta_{e1}\sinh^2\delta_{e2}\sinh^2\delta_e\}]d\phi d\psi\\
&\quad - \frac{4m\sin^2\theta}{\bar{\Delta}}[(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)(l_1\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e - l_2\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e)\\
&\quad + 2ml_2\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e]d\phi dt - \frac{4m\cos^2\theta}{\bar{\Delta}}[(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)\\
&\quad \times(l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e - l_1\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e)+2ml_1\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e]d\psi dt\\
&\quad + \frac{\sin^2\theta}{\bar{\Delta}}[(r^2+2m\sinh^2\delta_e+l_1^2)(r^2+2m\sinh^2\delta_{e1}+l_1^2\cos^2\theta+l_2^2\sin^2\theta)(r^2+2m\sinh^2\delta_{e2}\\
&\quad + l_1^2\cos^2\theta+l_2^2\sin^2\theta)+2m\sin^2\theta\{(l_1^2\cosh^2\delta_m - l_2^2\sinh^2\delta_m)(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)\\
&\quad + 4ml_1l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e - 2m\sinh^2\delta_{e1}\sinh^2\delta_{e2}\\
&\quad \times(l_1^2\cosh^2\delta_e+l_2^2\sinh^2\delta_e)-2ml_2^2\sinh^2\delta_e(\sinh^2\delta_{e1}+\sinh^2\delta_{e2})\}]d\phi^2\\
&\quad + \frac{\cos^2\theta}{\bar{\Delta}}[(r^2+2m\sinh^2\delta_e+l_2^2)(r^2+2m\sinh^2\delta_{e1}+l_1^2\cos^2\theta+l_2^2\sin^2\theta)(r^2+2m\sinh^2\delta_{e2}\\
&\quad + l_1^2\cos^2\theta+l_2^2\sin^2\theta)+2m\cos^2\theta\{(l_2^2\cosh^2\delta_e - l_1^2\sinh^2\delta_e)(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)\\
&\quad + 4ml_1l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e - 2m\sinh^2\delta_{e1}\sinh^2\delta_{e2}\\
&\quad \times(l_1^2\sinh^2\delta_e+l_2^2\cosh^2\delta_e)-2ml_1^2\sinh^2\delta_e(\sinh^2\delta_{e1}+\sinh^2\delta_{e2})\}]d\psi^2\Bigg],
\end{aligned}
\tag{18}
$$

where

$$
\begin{aligned}
\bar{\Delta} &\equiv (r^2+2m\sinh^2\delta_{e1}+l_1^2\cos^2\theta+l_2^2\sin^2\theta)(r^2+2m\sinh^2\delta_{e2}+l_1^2\cos^2\theta+l_2^2\sin^2\theta)\\
&\quad \times(r^2+2m\sinh^2\delta_e+l_1^2\cos^2\theta+l_2^2\sin^2\theta),
\end{aligned}
\tag{19}
$$

and the subscript E in the line element denotes the Einstein-frame. (In Eq. (15), it was not necessary to distinguish between the string- and the Einstein-frames since in the case of the Kerr solution the dilaton is zero.) The above solutions are obtained from the corresponding three-dimensional boosted solutions by first imposing the duality transformation (11) and then by constructing the five-dimensional fields, using the field redefinitions in section IIA. The $U(1)$ charges $Q's$, the ADM mass M and the angular momenta $J's$ are given by:

$$
Q^{(1)}_1 = 2m\cosh\delta_{e1}\sinh\delta_{e1},\quad Q^{(2)}_1 = 2m\cosh\delta_{e2}\sinh\delta_{e2},\quad Q = 2m\cosh\delta_e\sinh\delta_e,
$$

$$
\begin{aligned}
M &= 2m(\cosh^2\delta_{e1} + \cosh^2\delta_{e2} + \cosh^2\delta_e) - 3m \\
&= \sqrt{m^2 + (Q_1^{(1)})^2} + \sqrt{m^2 + (Q_1^{(2)})^2} + \sqrt{m^2 + Q^2}, \\
J_\phi &= 4m(l_1\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e - l_2\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e), \\
J_\psi &= 4m(l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e - l_1\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e).
\end{aligned} \tag{20}
$$

Note that solution has the inner and outer horizons at:

$$
r_\pm^2 = m - \frac{1}{2}l_1^2 - \frac{1}{2}l_2^2 \pm \frac{1}{2}\sqrt{(l_1^2 - l_2^2)^2 + 4m(m - l_1^2 - l_2^2)}, \tag{21}
$$

provided $m \geq (|l_1| + |l_2|)^2$.

The generating solution effectively corresponds to a six-dimensional target space background. It would be interesting to map this solution onto a σ-model action with the corresponding six-dimensional target space specified by the background fields (18).

The above solution with $Q = Q_1^{(1)} = Q_1^{(2)}$, *i.e.*, $\delta_e = \delta_{e1} = \delta_{e2}$, corresponds to the special case where both the dilaton φ and the internal-circle-modulus field g_{11} are constant. The solution with $Q = Q_1^{(2)}$, *i.e.*, $\delta_e = \delta_{e2}$, corresponds to the case where the six-dimensional dilaton $\varphi_6 = \varphi + \frac{1}{2}\log\det g_{11}$ is constant. In this case, with a subsequent rescaling of the asymptotic values of the scalar fields one obtains the static solutions studied in Ref. [5] and rotating solutions studied in Ref. [6].

C. T-Duality Transformations

The subset of the $[SO(5)\times SO(21)]/[SO(4)\times SO(20)]$ transformations yields the following charge assignments:

$$
\vec{\mathcal{Q}}' = \frac{1}{\sqrt{2}}\mathcal{U}^T \begin{pmatrix} U_5(\mathbf{e}_u - \mathbf{e}_d) \\ U_{21}\begin{pmatrix} \mathbf{e}_u + \mathbf{e}_d \\ 0_{16} \end{pmatrix} \end{pmatrix}, \tag{22}
$$

where

$$
\mathbf{e}_u^T \equiv (Q_1^{(1)}, \overbrace{0, ..., 0}^{4}), \quad \mathbf{e}_d^T \equiv (Q_1^{(2)}, \overbrace{0, ..., 0}^{4}), \tag{23}
$$

where $U_5 \in SO(5)/SO(4)$, $U_{21} \in SO(21)/SO(20)$, 0_{16} is a (16×1)-matrix with zero entries, and $\mathcal{U} \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}I_5 & -\frac{1}{\sqrt{2}}I_5 & 0 \\ \frac{1}{\sqrt{2}}I_5 & \frac{1}{\sqrt{2}}I_5 & 0 \\ 0 & 0 & I_{16} \end{pmatrix}$. The moduli field matrix M is transformed to

$$
M' = \mathcal{U}^T \begin{pmatrix} U_5 & 0 \\ 0 & U_{21} \end{pmatrix} \mathcal{U} M \mathcal{U}^T \begin{pmatrix} U_5^T & 0 \\ 0 & U_{21}^T \end{pmatrix} \mathcal{U}, \tag{24}
$$

The subsequent transformation by a constant $O(5,21)\times SO(1,1)$ matrix allows one to write the solution in terms of the arbitrary asymptotic values M_∞ and φ_∞.

IV. BPS-SATURATED LIMIT AND DEVIATIONS FROM IT

A. BPS-Saturated Limit

The BPS-limit (of the generating solution) is obtained by taking $m \to 0$, while keeping the three charges $Q_1^{(1)}$, $Q_1^{(2)}$ and Q, as well as J_ϕ and J_ψ finite. This is achieved by taking the following limits: $m \to 0$, $l_{1,2} \to 0$ and $\delta_{e1,e2,e} \to \infty$ while keeping $\frac{1}{2}me^{2\delta_{e1}} = Q_1^{(1)}$, $\frac{1}{2}me^{2\delta_{e2}} = Q_1^{(2)}$, $\frac{1}{2}me^{2\delta_e} = Q$, $l_1/m^{1/2} = L_1$, and $l_2/m^{1/2} = L_2$ constant [11]. The BPS-saturated solution is then of the following form:

$$
\begin{aligned}
A^{(1)}_{t\,1} &= \frac{\frac{1}{2}Q_1^{(1)}}{r^2+Q_1^{(1)}}, \quad A^{(1)}_{\phi\,1} = \frac{\frac{1}{4}J\sin^2\theta}{r^2+Q_1^{(1)}}, \quad A^{(1)}_{\psi\,1} = \frac{\frac{1}{4}J\cos^2\theta}{r^2+Q_1^{(1)}},\\
A^{(2)}_{t\,1} &= \frac{\frac{1}{2}Q_1^{(2)}}{r^2+Q_1^{(2)}}, \quad A^{(2)}_{\phi\,1} = \frac{\frac{1}{4}J\sin^2\theta}{r^2+Q_1^{(2)}}, \quad A^{(2)}_{\psi\,1} = \frac{\frac{1}{4}J\cos^2\theta}{r^2+Q_1^{(2)}},\\
B_{t\phi} &= -\frac{\frac{1}{2}J\sin^2\theta(r^2+\frac{1}{2}Q_1^{(1)}+\frac{1}{2}Q_1^{(2)})}{(r^2+Q_1^{(1)})(r^2+Q_1^{(2)})}, \qquad B_{t\psi} = \frac{\frac{1}{2}J\cos^2\theta(r^2+\frac{1}{2}Q_1^{(1)}+\frac{1}{2}Q_1^{(2)})}{(r^2+Q_1^{(1)})(r^2+Q_1^{(2)})},\\
B_{\phi\psi} &= \frac{Q\cos^2\theta\sin^2\theta(r^2+\frac{1}{2}Q_1^{(1)}+\frac{1}{2}Q_1^{(2)})}{(r^2+Q_1^{(1)})(r^2+Q_1^{(2)})}, \quad g_{11} = \frac{r^2+Q_1^{(1)}}{r^2+Q_1^{(2)}}, \quad e^{\varphi} = \frac{(r^2+Q)}{[(r^2+Q_1^{(1)})(r^2+Q_1^{(2)})]^{\frac{1}{2}}},\\
ds_E^2 &= \bar{\Delta}^{\frac{1}{3}}\Bigg[-\frac{r^4}{\bar{\Delta}}dt^2+\frac{dr^2}{r^2}+d\theta^2+\frac{J^2\cos^2\theta\sin^2\theta}{2\bar{\Delta}}d\phi d\psi-\frac{2Jr^2\sin^2\theta}{\bar{\Delta}}dtd\phi+\frac{2Jr^2\cos^2\theta}{\bar{\Delta}}dtd\psi\\
&\quad+\frac{\sin^2\theta}{\bar{\Delta}}\{(r^2+Q_1^{(1)})(r^2+Q_1^{(2)})(r^2+Q)-\frac{1}{4}J^2\sin^2\theta\}d\phi^2\\
&\quad+\frac{\cos^2\theta}{\bar{\Delta}}\{(r^2+Q_1^{(1)})(r^2+Q_1^{(2)})(r^2+Q)-\frac{1}{4}J^2\cos^2\theta\}d\psi^2\Bigg],
\end{aligned}
\tag{25}
$$

where

$$\bar{\Delta} \equiv (r^2+Q_1^{(1)})(r^2+Q_1^{(2)})(r^2+Q). \tag{26}$$

The solution is specified by three charges and *only one* rotational parameter J [12]:

$$J_\phi = -J_\psi \equiv J = (2Q_1^{(1)}Q_1^{(2)}Q)^{\frac{1}{2}}(L_1-L_2), \tag{27}$$

while its ADM mass saturates the Bogomol'nyi bound:

$$M_{BPS} = Q_1^{(1)} + Q_2^{(2)} + Q. \tag{28}$$

---

[11] Since we choose the boost parameters to be positive, *i.e.*, $\delta \to +\infty$, the $U(1)$ charges are positive. Had we taken the boost parameters to be negative, the $U(1)$ charges are negative but they will appear with absolute values in the solutions.

[12] In the case one takes one or three boost parameters to be negative, one obtains the BPS limit with $J_\phi = J_\psi$.

The surface area of the horizon is of the form:

$$A_{BPS} = 4\pi^2[(Q_1^{(1)}Q_1^{(2)}Q)(1-\frac{1}{2}(L_1-L_2)^2)]^{\frac{1}{2}} = 4\pi^2[Q_1^{(1)}Q_1^{(2)}Q - \frac{1}{4}J^2]^{\frac{1}{2}}. \tag{29}$$

The above solution with $Q = Q_1^{(1)} = Q_1^{(2)}$ and the subsequently rescaled asymptotic value of the dilaton $\varphi_\infty \neq 0$ was obtained for the static BPS-saturated states in Ref. [2] and for rotating ones in Ref. [4]. The above general form of the solution (with all three charges different) was given in Ref. [11] as a solution of the corresponding conformal σ-model [13].

The mass of the BPS-saturated solution (at generic points of the moduli and the coupling space) can be obtained by acting with the subset of $[SO(5)\times SO(21)]/[SO(4)\times SO(20)]$ transformations and then undoing the asymptotic values of moduli $M_\infty = I_{26}$ and the dilaton $\varphi_\infty = 0$ with the constant $O(5,21)\times SO(1,1)$ transformations, as spelled out in Section III A. The procedure yields the following form of the ADM mass

$$M_{BPS} = e^{2\varphi_\infty/3}[\vec{\alpha}^T(\mathcal{M}_\infty + L)\vec{\alpha}]^{1/2} + e^{-4\varphi_\infty/3}\beta, \tag{30}$$

while the macroscopic entropy $\mathbf{S} = A/(4G_N^{D=5})$ (here A is the area of the horizon at $r_+ = r_- = 0$) is of the form:

$$\mathbf{S}_{BPS} = 4\pi\sqrt{\beta(\vec{\alpha}^T L\vec{\alpha}) - \frac{1}{4}J^2}, \tag{31}$$

where the expression has been written in terms of conserved, quantized charges β and $\vec{\alpha}$, which are related to the physical charges Q and $\vec{\mathcal{Q}}$ (22) in the following way:

$$Q = e^{-4\varphi_\infty/3}\beta, \qquad \vec{\mathcal{Q}} = e^{2\varphi_\infty/3}M_\infty\vec{\alpha}. \tag{32}$$

While the mass depends on the asymptotic values of moduli and the dilaton coupling, the entropy for the BPS states is a universal quantity [9,10,20,21], and thus depends only on conserved (quantized) charges β and $\vec{\alpha}$.

B. Deviations from the BPS-Saturated Limit

It is useful to obtain the explicit form of the solution that is infinitesimally close to the BPS limit, since in certain specific cases the microscopic entropy can be calculated for such solutions as well [5,6].

The deviation from the BPS limit of the generating solution is achieved by taking m small, but non-zero, while keeping other parameters $Q_1^{(1)}, Q_1^{(2)}, Q, L_1$, and L_2 finite.

We have pursued the expansion of the solution up to the linear order in m. In particular, now, the solution has the inner and outer horizons at:

---

[13] Related σ-models, describing more general 5-dimensional rotating solutions, are studied in Ref. [19].

$$r_\pm^2 = m\left(1 - \frac{1}{2}(L_1^2 + L_2^2) \pm \frac{1}{2}\sqrt{[2-(L_1+L_2)^2][2-(L_1-L_2)^2]}\right). \tag{33}$$

The area of the outer horizon r_+ can be written as:

$$\begin{aligned} A = 4\pi^2 \Bigg[&(Q_1^{(1)} Q_1^{(2)} Q)\sqrt{1 - \frac{1}{2}(L_1 - L_2)^2} + m(Q_1^{(1)} Q_1^{(2)} + Q_1^{(1)} Q + Q_1^{(2)} Q) \\ &\times \sqrt{[1 - \frac{1}{2}(L_1 + L_2)^2]} \Bigg]^{\frac{1}{2}}, \end{aligned} \tag{34}$$

which is correct in the expansion up to the leading order in m, only. The rotational parameters J_ϕ and J_ψ are not equal in magnitude and opposite in sign anymore, but are of the form:

$$\begin{aligned} J &\equiv \tfrac{1}{2}(J_\phi - J_\psi) = (2Q_1^{(1)} Q_1^{(2)} Q)^{\frac{1}{2}}(L_1 - L_2) + \mathcal{O}(m^2), \\ \Delta J &\equiv \tfrac{1}{2}(J_\psi + J_\phi) = m(2Q_1^{(1)} Q_1^{(2)} Q)^{\frac{1}{2}}\left(\frac{1}{Q_1^{(1)}} + \frac{1}{Q_1^{(2)}} + \frac{1}{Q}\right)(L_1 + L_2) + \mathcal{O}(m^2), \end{aligned} \tag{35}$$

while the ADM mass is still conveniently kept as:

$$M = \sqrt{m^2 + (Q_1^{(1)})^2} + \sqrt{m^2 + (Q_1^{(2)})^2} + \sqrt{m^2 + Q^2}. \tag{36}$$

Note that in the case where one of the charges is taken small, *e.g.*, $Q_1^{(1)} \to 0$, as in the recent study of the microscopic entropy near the BPS-saturated limit [5,6], the ADM mass is $M = M_{BPS} + \mathcal{O}(m)$, while the area is $A = \mathcal{O}(m^{1/2})$. However, when all the charges are taken non-zero the deviation from the BPS limit is of the form $M = M_{BPS} + \mathcal{O}(m^2)$ and $A = A_{BPS} + \mathcal{O}(m)$. In particular, further study of the microscopic entropy for an infinitesimal deviation from the general BPS-saturated limit is needed.

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REFERENCES

[1] J. Polchinski, S. Chaudhuri and C.V. Johnson, NSF-ITP-96-003, hep-th/9602052.

[2] A. Strominger and C. Vafa, HUTP-96-A002, hep-th/9601029.

[3] C.G. Callan and J.M. Maldacena, PUPT-1591, hep-th/9602043.

[4] J.C. Breckenridge, R.C. Myers, A.W. Peet and C. Vafa, HUTP-96-A005, hep-th/9602065.

[5] G.T. Horowitz and A. Strominger, hep-th/9602051.

[6] J.C. Breckenridge, D.A. Lowe, R.C. Myers, A.W. Peet, A. Strominger and C. Vafa, HUTP-96/A008,hep-th/9603078.

[7] J.M. Maldacena and A. Stromginer, hep-th/9603060.

[8] C.V. Johnson, R.R. Khuri and R.C. Myers, CERN-TH-96-62, hep-th/9603061.

[9] F. Larsen and F. Wilczek, PUPT-1576, hep-th/9511064.

[10] M. Cvetič and A.A. Tseytlin, IASSNS-HEP-95/102, hep-th/ 9512031, to be published in Phys. Rev. **D**.

[11] A.A. Tseytlin, IMPERIAL-TP-95-96-22, hep-th/9601177, to be published in Modern Physics Letters.

[12] W. Israel, Phys. Rev. **164** (1967) 1776; W. Israel, Cummun. Math. Phys. **8** (1968) 245; B. Carter, Phys. Rev. Lett. **26** (1970) 331; S.W. Hawking, Phys. Rev. Lett. **26** (1971) 1334; S.W. Hawking, Cummun. Math. Phys. **25** (1972) 152.

[13] J. Maharana and J.H. Schwarz, Nucl. Phys. **B390** (1993) 3, hep-th/9207016.

[14] A. Sen, Int. J. Mod. Phys. **A9** (1994) 3707, hep-th/9402002.

[15] C.M. Hull and P.K. Townsend, Nucl. Phys. **B438** (1995) 109.

[16] A. Sen, Nucl. Phys. **B434** (1995) 179, hep-th/9408083.

[17] A. Sen, Nucl. Phys. **B440** (1995) 421.

[18] R.C. Myers and M. Perry, Ann. Phys. **172** (1986) 304.

[19] A.A. Tseytlin, IMPERIAL-TP-95-96-32 preprint (1996) hep-th/9603099.

[20] A. Strominger, hep-th/9602111.

[21] S. Ferrara and R. Kallosh, UCLA-96-TEP-8, hep-th/9602136 and CERN-TH/96-66, hep-th/9603090.